The Connected Caveman ~~Problem~~ Graph Model

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CSL7390 Social Network Analysis Lectures 35
22nd April 2024

Historical background

After proposing the Watts-Strogatz model in 1998, Duncan J. Watts successfully defended his PhD thesis under the guidance of Prof. Steven Strogatz at the Cornell University.

Subsequently, Watts started his career at the Santa Fe Institute and wrote a paper with two objectives in mind:

- Formalize the definition of a small-world graph
- Propose a set of graph generator algorithms that can generate model small-world graphs

Prerequisites

Characteristic path length (L) = Average distance between any two vertices.

(It is a global measure since the structure of the whole graph needs to be known in order to calculate L.)

Clustering coefficient of a vertex (C_v) = What fraction of my friends are friends among themselves (a local measure).

Clustering coefficient of a graph (C) = Average of the clustering coefficients of all vertices (a global measure).

Formal definition of a small-world graph [Watts 1999]

A small-world graph is a graph that satisfies the following properties:

- A large number of vertices (n)
- Sparsely connected ($n >> k_{max}$) and decentralized ($k_{max} >> 1$) where k_{max} = Maximum degree
- Characteristic path length (L) is almost equal to that of a random graph ($L \approx L_{random}$) meaning L should remain small even when n grows large
- Cluster coefficient much greater than that of a random graph ($C >> C_{random}$)

The connected caveman graph model [Watts 1999]

It has the following properties:

- A large number of vertices (n)
- Sparsely connected ($n >> k_{max}$) and decentralized ($k_{max} >> 1$) where k_{max} = Maximum degree
- **$L >> L_{random}$ meaning L grows large when n grows large**
- Cluster coefficient much greater than that of a random graph ($C >> C_{random}$)

Hence, the connected caveman graph model is **not a small-world graph model**.

Shortcut-based Φ-model [Watts 1999]

It satisfies the following properties:

- A large number of vertices (n)
- Sparsely connected ($n >> k_{max}$) and decentralized ($k_{max} >> 1$) where k_{max} = Maximum degree
- Characteristic path length (L) is almost equal to that of a random graph ($L \approx L_{random}$) meaning L should remain small even when n grows large
- Cluster coefficient much greater than that of a random graph ($C >> C_{random}$)

Hence, it is a small-world graph model.

Contraction-based Ψ-model [Watts 1999]

It satisfies the following properties:

- A large number of vertices (n)
- Sparsely connected ($n >> k_{max}$) and decentralized ($k_{max} >> 1$) where k_{max} = Maximum degree
- Characteristic path length (L) is almost equal to that of a random graph ($L \approx L_{random}$) meaning L should remain small even when n grows large
- Cluster coefficient much greater than that of a random graph ($C >> C_{random}$)

Hence, it is a small-world graph model.

References

- Watts, ‘Networks, Dynamics, and the Small-World Phenomenon’, American Journal of Sociology, 1999: https://doi.org/10.1086/210318

Optional references

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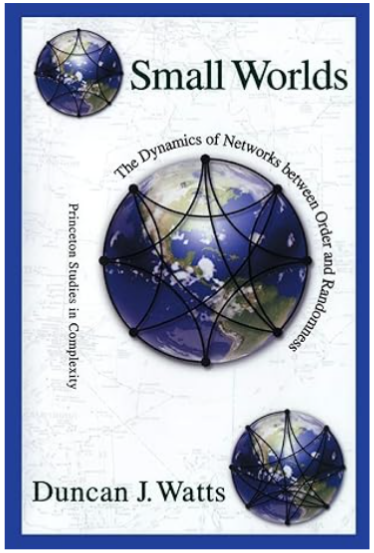

Small Worlds: The Dynamics of Networks between Order and Randomness: 36 (Princeton Studies in Complexity)

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Optional references(contd.)

- https://mathworld.wolfram.com/CavemanGraph.html
- https://networkx.org/documentation/stable/reference/generated/networkx.generators.community.connected_caveman_graph.html

Thank you