Network Models

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What we discussed in the last module

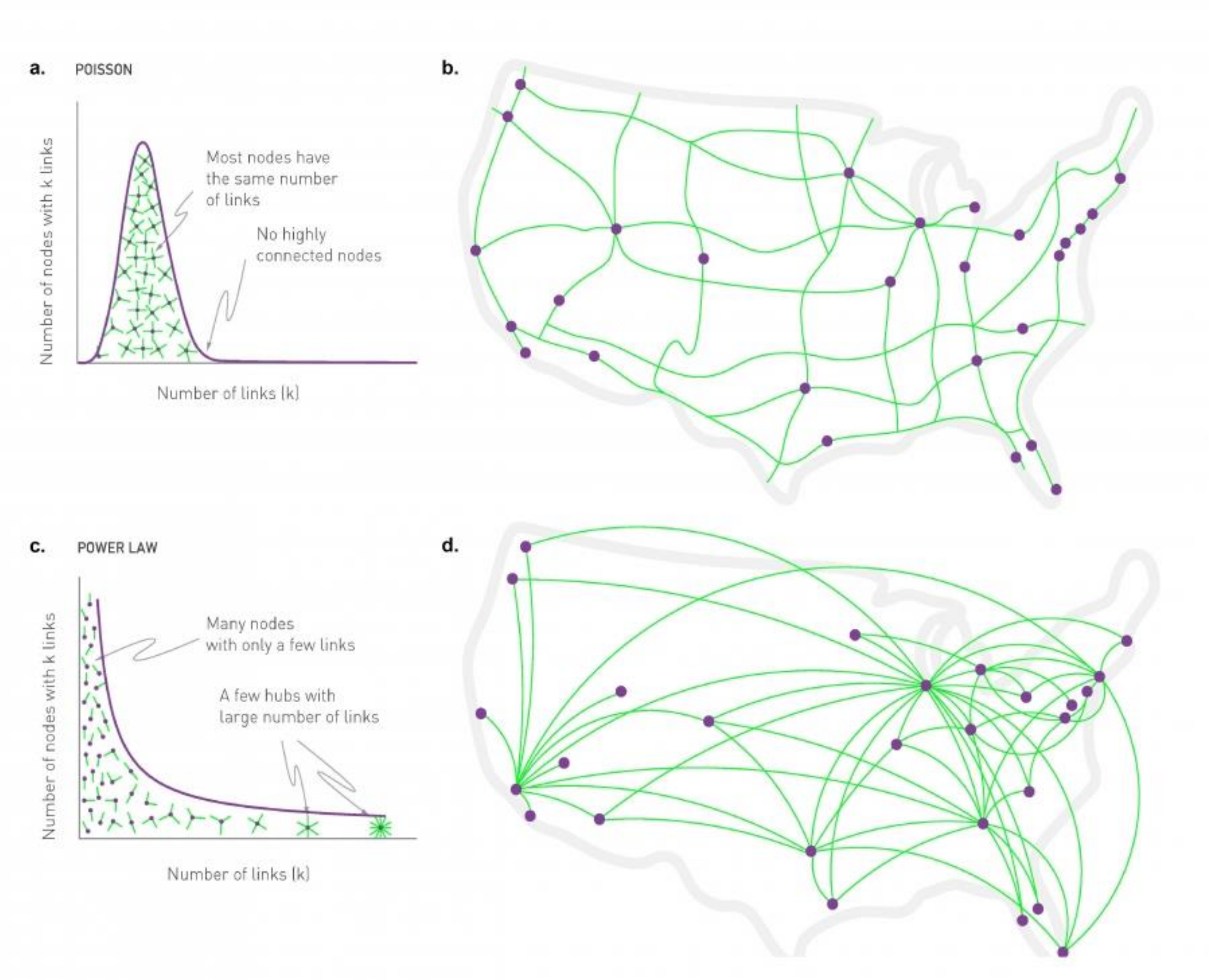
Social groups

- How many types of groups are there?
- How does groupism affect the balance of the overall social structure?
- How does a social actor choose its group(s)?

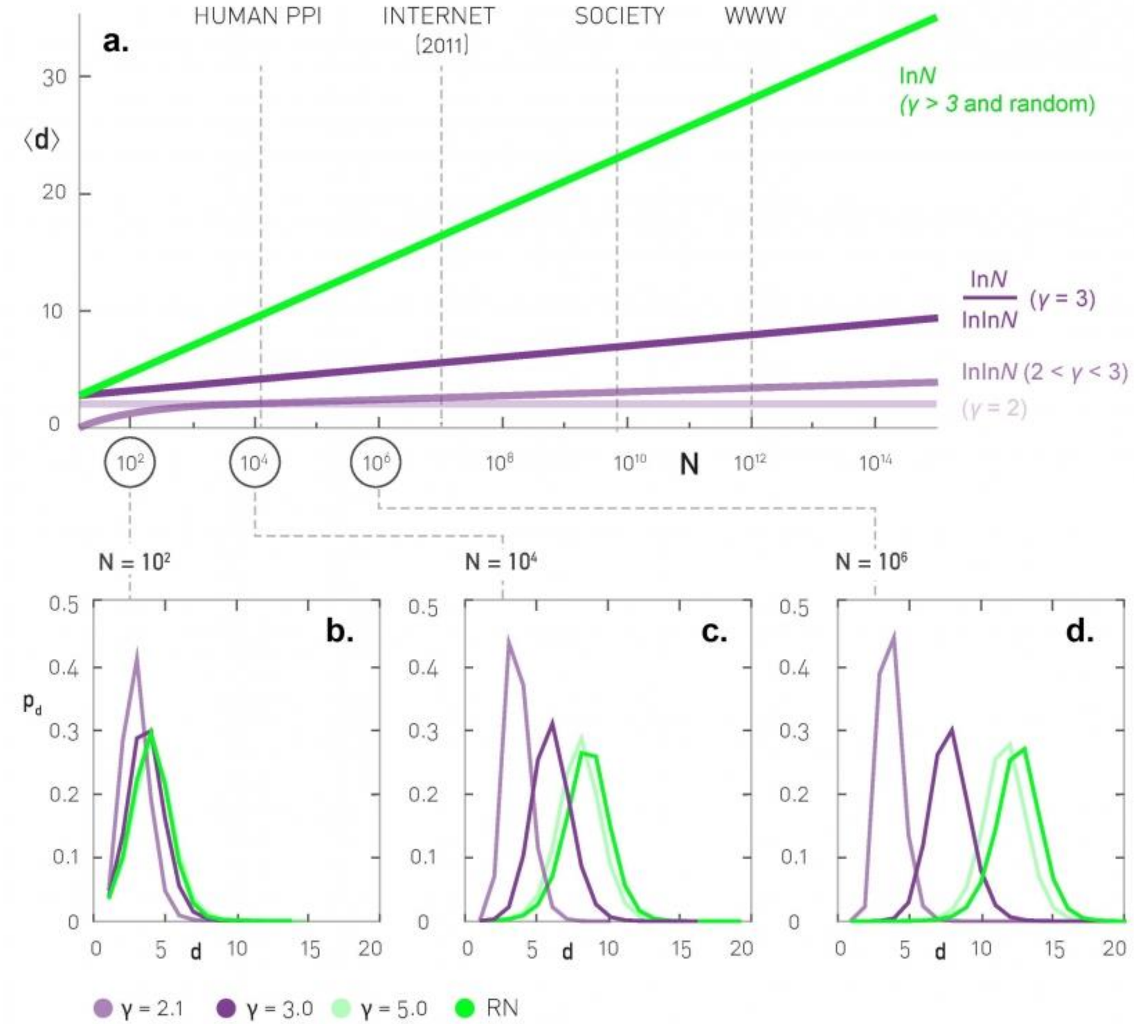
Important notation

P_k = The probability that a randomly chosen node has exactly k neighbours.

Random networks vs. scale-free networks



Effect of degree exponent on distance in scale-free networks



$\langle d \rangle \propto \gamma$

Small world

Critical point

Ultra-small world

Hub and spoke

p_d
= Distance distribution
= Given a value of d, what percentage of distances have that value of d

p_d is more sensitive to γ for higher values of N

The Barabasi-Albert model

- **Growth:** Vertex set grows with time.

- **Preferential attachment:** The probability that a newly added vertex will connect to an existing vertex i is as follows:

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

Barabási and Albert, Science, 1999

The Barabasi-Albert model leads to a scale-free network

- Barabasi and Albert showed that growth along with preferential attachment lead to a scale-free network.

- In theory, it is possible for scale-free networks to emerge through mechanisms different from the Barabasi-Albert model.

- However, most real-world networks exhibit preferential attachment. It suggests that they have emerged following the Barabasi-Albert model.

Barabási and Albert, Science, 1999

Limitations of the Barabasi-Albert model

- New edges between already existing vertices
- Disappearance of edges
- Disappearance of vertices
- Directed networks
- Prior weights of vertices emerging from non-network factors

- Therefore, the B-A model does not present how the real-world networks had emerged.
  Instead, this model presents one of the mechanisms through which the real-world networks might have gained their scale-free property.

The fuzzy granular social network (FGSN) Model

X = The universe of elements.
A fuzzy set A is an ordered pair (X, μ_A)
$= \{(x, \mu_A(x) \,|x \in X)\}$
where
μ_A = The fuzzy set membership function and
$\mu_A(x)$ = The degree of membership of element x to fuzzy set A.

Suppose, $X = \{x_1, x_2, x_3\}$ and $\mu_A = \{0.5, 0, 0.1\}$.
Then $\mu_A(x_1) = 0.5$.

Cardinality of a fuzzy set $|A|$
$= \sum_{x \in X} \mu_A(x)$

Zadeh, Information Control, 1965

The FGSN Model (contd.)

An FGSN model $\mathcal{S}$
$= (\mathcal{C}, \mathcal{V}, \mathcal{G})$
where
$\mathcal{V}$ = The set of all vertices,
$\mathcal{C}$ = The set of all granule centres,
$\mathcal{G}$ = The set of all granules.
$\mathcal{G} = \{\bigcup \phi(c) | c \in \mathcal{C}\}$ where
$\phi(c)$ = The granule centred around vertex c.
The degree of membership of vertex v in the granule centred around vertex c

$$\mu_c(v) = \begin{cases} 0 & \text{if } d(c, v) > r \\ \frac{1}{1+d(c,v)} & \text{otherwise} \end{cases}$$

The FGSN Model for directed graphs

An FGSN model $\mathcal{S}$ can be extended to directed graphs
$= (\mathcal{C}, \mathcal{V}, \mathcal{G}_{IN}, \mathcal{G}_{OUT})$
where
$\mathcal{G}_{IN}$ = The granules defined based on the incoming edges of the vertices,
$\mathcal{G}_{OUT}$ = The granules defined based on the outgoing edges of the vertices.

References

- ‘Network Science’ by Albert-László Barabási, a freely available textbook online: http://networksciencebook.com/
  - Chapter 3 ‘Random Networks’
  - Chapter 4 ‘The Scale-Free Property’
  - Chapter 5 ‘The Barabási-Albert Model’
- Fuzzy Granular Social Networks
  - Kundu and Pal, 2015, Information Sciences: http://dx.doi.org/10.1016/j.ins.2015.03.065

Thank you